1013 ECEB: 10am A-R
1015 ECEB: 10am S-Z, 9am A-H
2017 ECEB: 9am I-Z

Course Review

12/18/18 ⇒ 1:30-4:30pm (Main)
12/19/18 ⇒ 1:30-4:30pm (conflict)

Course Review:

Chapter 2: $V = V_m \cos(\omega t + \theta_v)$ $\qquad I = I_m \cos(\omega t + \theta_i)$

$\bar{V} = \frac{V_m}{\sqrt{2}}\angle\theta_v = V_{rms}\angle\theta_v$ $\qquad \bar{I} = \frac{I_m}{\sqrt{2}}\angle\theta_i = I_{rms}\angle\theta_i$

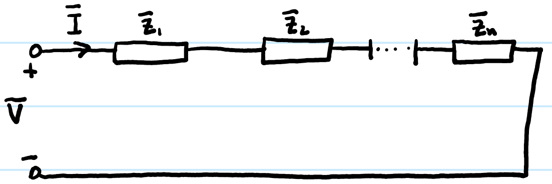
$\bar{S} = \bar{V}\bar{I}^* \Rightarrow \bar{S} = (V_{rms}\angle\theta_v)(I_{rms}\angle -\theta_i) \Rightarrow \bar{S} = V_{rms}I_{rms}\angle\theta_v - \theta_i = P + jQ$

$\theta_{PF} = \theta_v - \theta_i$ $\qquad PF = \cos(\theta_{PF})$

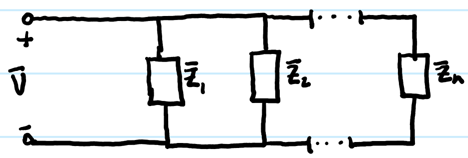
$\theta_{PF} > 0$: lagging PF $\qquad \theta_{PF} < 0$: leading PF

$S = V_{rms}I_{rms}$ $\qquad P = S\cos(\theta_{PF})$ $\qquad Q = S\sin(\theta_{PF})$

$S = \sqrt{P^2 + Q^2}$ $\qquad \theta_{PF} = \tan^{-1}\left(\frac{Q}{P}\right)$

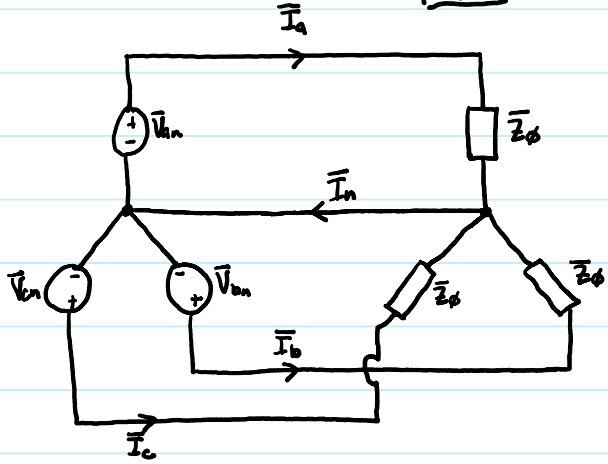


$\bar{S}_{TOT} = \bar{S}_1 + \bar{S}_2 + \ldots + \bar{S}_n$

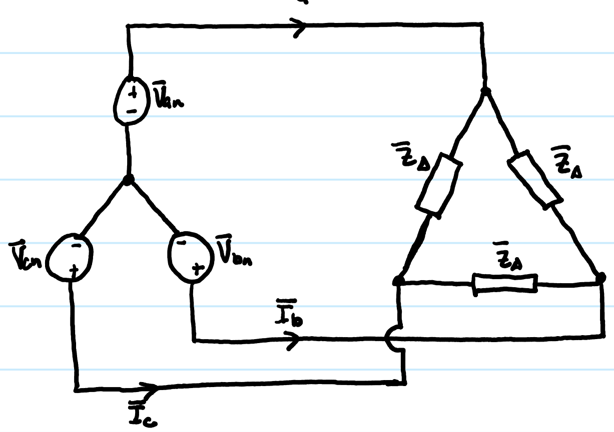


$\bar{S}_{tot} = \bar{S}_1 + \bar{S}_2 + \ldots + \bar{S}_n$

Wye



Delta



$\bar{I}_\phi = \bar{I}_L \qquad \bar{I}_n = 0$

$\bar{V}_{an} = V_\phi\angle 0$ $\qquad \bar{V}_{ab} = \sqrt{3}V_\phi\angle 30°$ $\qquad \bar{V}_{ab} = V_L\angle 0°$ $\qquad \bar{I}_a = \sqrt{3}I_\phi\angle -30-\theta$

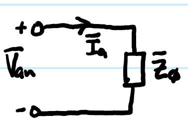
$\bar{V}_{bn} = V_\phi\angle -120°$ $\qquad \bar{V}_{bc} = \sqrt{3}V_\phi\angle -90°$ $\qquad \bar{V}_{bc} = V_L\angle -120°$ $\qquad \bar{I}_b = \sqrt{3}I_\phi\angle -150-\theta$

$\bar{V}_{cn} = V_\phi\angle 120°$ $\qquad \bar{V}_{ca} = \sqrt{3}V_\phi\angle 150°$ $\qquad \bar{V}_{ca} = V_L\angle 120°$ $\qquad \bar{I}_c = \sqrt{3}I_\phi\angle 90-\theta$

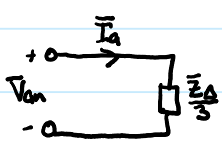
$\bar{S}_{3\phi} = 3\bar{V}_a\bar{I}_a^*$

$S_{3\phi} = 3V_aI_a = \sqrt{3}V_LI_L$ for both connections

Perphase equivalent circuit: Wye:



Delta:



Course Review

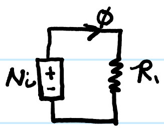
Chapter 3:

$$\oint_C \underline{H} \cdot d\underline{l} = \int_S \underline{J}_f \cdot \hat{n}\, dA \qquad \oint_S \underline{B} \cdot \hat{n}\, dA = 0$$

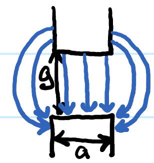
$$\sum H_n l_n = Ni \qquad B_1 A_1 = B_2 A_2 \qquad \text{Linear: } B_1 = \mu_1 H_1$$

Reluctance: $\mathcal{R}_1 = \dfrac{l_1}{\mu_1 A_1}$

MMF circuit:



Fringing:



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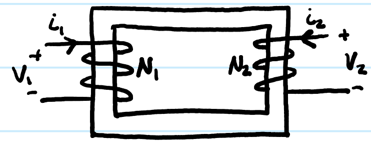
$$A_g = (a+g)(b+g) = ab\left(1 + g\left(\frac{1}{a} + \frac{1}{b}\right) + \frac{g^2}{ab}\right)$$

Flux linkage: $\lambda = N\phi \Rightarrow \lambda = Li$

$$V = \frac{d\lambda}{dt} \Rightarrow V = L\frac{di}{dt}$$

Mutual Inductance:

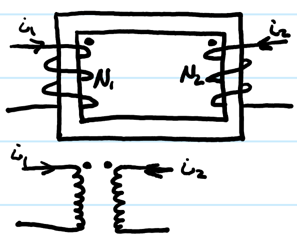


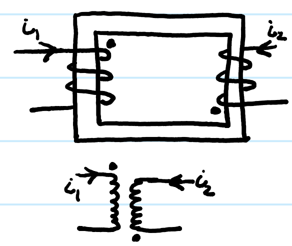
Coefficient of coupling: $k = \dfrac{M}{\sqrt{L_1 L_2}}$

$$\lambda_1 = L_1 i_1 + M i_2 \qquad \lambda_2 = M i_1 + L_2 i_2$$

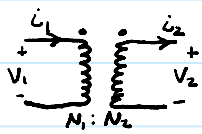
$$V_1 = \frac{d\lambda_1}{dt} \Rightarrow V_1 = L_1\frac{di_1}{dt} + M\frac{di_2}{dt} \qquad V_2 = \frac{d\lambda_2}{dt} \Rightarrow V_2 = M\frac{di_1}{dt} + L_2\frac{di_2}{dt}$$

Polarity Markings (Dot notation)





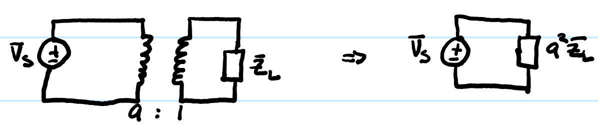
Ideal transformer:



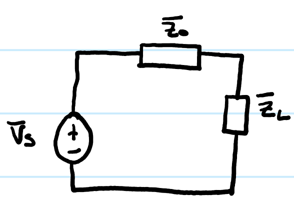
$$a = \frac{N_1}{N_2}$$

$$\frac{V_1}{V_2} = a \qquad \frac{i_1}{i_2} = \frac{1}{a}$$

Transformers and impedances:



Max power transfer:



$$\bar{Z}_L = \bar{Z}_o^*$$

$$|\bar{Z}_L| = |\bar{Z}_o| \quad \text{for } \frac{R_L}{X_L} = \text{Constant}$$

Course Review

Chapter 4: **Translation:** $\lambda = L(x)i$

$$V = \frac{d\lambda}{dt} \Rightarrow V = \underbrace{L\frac{di}{dt}}_{\text{transformer voltage}} + \underbrace{\frac{\partial L}{\partial x}\frac{dx}{dt}i}_{\text{speed voltage}}$$

Rotation: $\lambda_s = L_s i_s + M\cos(\theta) i_r$

$\lambda_r = M\cos(\theta) i_s + L_r i_r$

$$V_s = L_s\frac{di_s}{dt} + M\cos(\theta)\frac{di_r}{dt} - M\sin(\theta)\frac{d\theta}{dt}i_r$$

$$V_r = L_r\frac{di_r}{dt} + M\cos(\theta)\frac{di_s}{dt} - M\sin(\theta)\frac{d\theta}{dt}i_s$$

Energy: $W_m = \int_0^{\lambda} i(\hat{\lambda})\,d\hat{\lambda}$

Translation: $f^e = -\frac{\partial W_m}{\partial x}$, $i = \frac{\partial W_m}{\partial \lambda}$ **Rotation:** $T^e = -\frac{\partial W_m}{\partial \theta}$, $i = \frac{\partial W_m}{\partial \lambda}$

Co-energy: $W_m + W_m' = i\lambda$, $W_m' = \int_0^{i} \lambda(\hat{i})\,d\hat{i}$

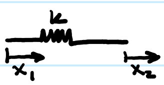
Translation: $f^e = \frac{\partial W_m'}{\partial x}$, $\lambda = \frac{\partial W_m'}{\partial i}$ **Rotation:** $T^e = \frac{\partial W_m'}{\partial \theta}$, $\lambda = \frac{\partial W_m'}{\partial i}$

Multiple input systems: $W_m' = \int_0^{i_1} \lambda_1(\hat{i}_1, i_2 = 0, x)\,d\hat{i}_1 + \int_0^{i_2} \lambda_2(i_1, \hat{i}_2, x)\,d\hat{i}_2$

Energy conservation: $\Delta W_{m\,a\to b} = EFE_{a\to b} + EFM_{a\to b}$ $\quad EFE_{a\to b} = \int_{\lambda_a}^{\lambda_b} i\,d\lambda$ $\quad EFM_{a\to b} = \int_{x_a}^{x_b} -f^e dx$

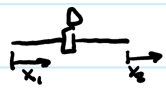
over cycle: $EFE_{cycle} + EFM_{cycle} = 0$

Springs:



$|f^s| = k(x_2 - x_1)$

Dampers:



$|f^d| = D(\dot{x}_2 - \dot{x}_1)$

State space: $m\ddot{x} = \Sigma F_x \Rightarrow$ $\frac{dx}{dt} = \dot{x}$, $\frac{d\dot{x}}{dt} = \frac{1}{m}(\Sigma F_x)$ $\Rightarrow \frac{dx_1}{dt} = x_2$, $\frac{dx_2}{dt} = \frac{1}{m}(\Sigma F_x)$

Euler's Method: $\frac{dx}{dt} = f(x,t) \Rightarrow x(t) = x(t-\Delta t) + \Delta t f(x(t-\Delta t), t-\Delta t)$ $\quad \Delta t \ll 1$

Chapter 5: **Linearization:** $\dot{\underline{x}} = f(\underline{x}, \underline{u}, t)$

$0 = f(\underline{x}_{eq}, \hat{\underline{u}}, t)$

$\underline{x} = \underline{x}_{eq} + \Delta\underline{x}$

$\underline{u} = \hat{\underline{u}} + \Delta\underline{u}$

$$J = \begin{bmatrix} \frac{\partial f_1}{\partial x_1} & \frac{\partial f_1}{\partial x_2} & \cdots & \frac{\partial f_1}{\partial x_n} \\ \frac{\partial f_2}{\partial x_1} & \frac{\partial f_2}{\partial x_2} & \cdots & \frac{\partial f_2}{\partial x_n} \\ \vdots & \vdots & & \vdots \\ \frac{\partial f_n}{\partial x_1} & \cdots & \cdots & \frac{\partial f_n}{\partial x_n} \end{bmatrix} \qquad B = \begin{bmatrix} \frac{\partial f_1}{\partial u_1} & \cdots & \frac{\partial f_1}{\partial u_m} \\ \frac{\partial f_2}{\partial u_1} & \cdots & \frac{\partial f_2}{\partial u_m} \\ \vdots & & \vdots \\ \frac{\partial f_n}{\partial u_m} & \cdots & \frac{\partial f_n}{\partial u_m} \end{bmatrix}$$

$$\Delta\dot{\underline{x}} = J\big|_{x_{eq}} \Delta\underline{x} + B\big|_{x_{eq}} \Delta\underline{u}$$

Stability of equilibrium point: $\det(J - \alpha I) = 0$

$Re\{\alpha_i\} < 0$ for all i's : stable

$Re\{\alpha_i\} > 0$ for 1 i : unstable

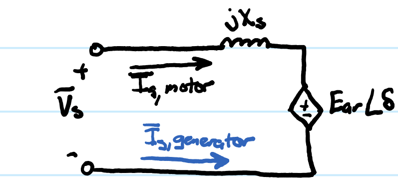
$Re\{\alpha_i\} = 0$: marginally stable

Course Review

Chapter 6: Synchronous machines

$\omega_m = \frac{2}{p}\omega_s$

$\omega_s = 2\pi f$



$E_{ar} = \frac{\omega_s M}{\sqrt{2}} I_r$

$P_T = 3\,\mathrm{Re}\{\bar{V}_s \bar{I}_s^*\} \Rightarrow P_T = \frac{3 V_s E_{ar}}{X_s}\sin(\delta)$

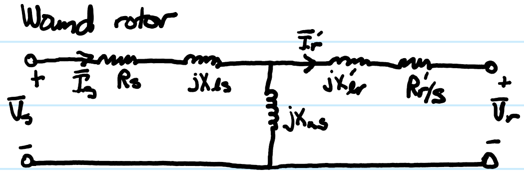
$T^e = \frac{P_T}{\omega_m}$

Chapter 7: Induction machines

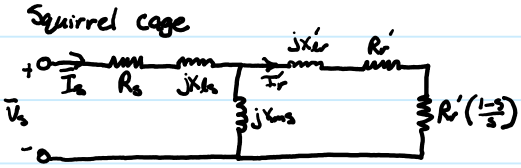
- * 2 phase and 3 phase generate rotating magnetic fields
- * Maxwell's equations: changing magnetic field induces an electric field (V)
- * Voltage creates a current inside conductors on rotor
- * Lenz's law: induced currents flow in the direction to counter changing applied magnetic field
- * Lorentz force: Force between a current and magnetic field causes rotor to rotate.

slip: $s = \frac{\omega_s - \left(\frac{P}{2}\right)\omega_m}{\omega_s}$

Wound rotor



Squirrel cage



$P_{AG} = 3|\bar{I}_r'|^2 \frac{R_r'}{s}$

$P_m = (1-s)P_{AG}$

$T^e = \frac{P_m}{\omega_m} = \frac{P_{AG}}{\frac{2}{p}\omega_s}$